Calc BC §9-5 Pg 523 # 8-17

(8) $\sum_{n=1}^{\infty} \frac{3}{n^{1/2}}$ P-series $p < 1$, diverges

or $\lim_{b \to \infty} \int_1^b 3x^{-1/2}\,dx$

$= \lim_{b \to \infty} 6x^{1/2}\Big]_1^b$

$= \lim_{b \to \infty} 6\sqrt{b} - 6 = \infty$, diverges

(9) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$ compare to $\sum \frac{1}{n}$

$\lim_{n \to \infty} \frac{\ln(n)}{n} \cdot \frac{n}{1}$

$= \lim_{n \to \infty} \ln(n) = \infty$

since $\sum \frac{1}{n}$ diverges so does $\sum \frac{\ln(n)}{n}$ by LTC.

(could also have used DCT)

(10) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

$\lim_{b \to \infty} \int_1^{\infty} \frac{1}{2x-1}\,dx$ $\quad u = 2x-1$, $du = 2\,dx$

$= \lim_{b \to \infty} \frac{1}{2}\int_1^{\infty} \frac{1}{u}\,du$

$= \lim_{b \to \infty} \frac{1}{2}\ln u\Big]_1^{\infty} = \infty$

diverges by Integral Test

(11) $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$

geometric w/ $r = \frac{1}{\ln 2}$

$\approx 1.44 > 1$

diverges.

or Ratio Test or . . . .

$\lim_{n \to \infty} \frac{1}{(\ln 2)^{n+1}} \cdot \frac{(\ln 2)^n}{1}$

$= \frac{1}{\ln 2} > 1$

diverges

(12) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$

geometric w/

$r = \frac{1}{\ln 3} < 1$

$\therefore$ converges

(Ratio Test is also nice)

(13) $\displaystyle\sum_{n=1}^{\infty} n \cdot \sin\left(\frac{1}{n}\right)$

$\lim_{n\to\infty} n \cdot \sin\left(\frac{1}{n}\right) \qquad u = \frac{1}{n}$

$= \lim_{u\to 0} \frac{\sin u}{u} = 1 \neq 0$

diverges by n^{th} term

(15) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

Compare to:

$\sum \frac{\sqrt{n}}{n^2}$

$= \sum \frac{1}{n^{3/2}}$

p-series

$p > 1$, converges

since

$\frac{\sqrt{n}}{n^2+1} < \frac{\sqrt{n}}{n^2}$

and $\sum \frac{\sqrt{n}}{n^2}$ converges (see ↑)

$\sum \frac{\sqrt{n}}{n^2+1}$ converges

by DCT

(14) $\displaystyle\sum_{n=0}^{\infty} \frac{e^n}{1+e^{2n}}$

$\lim_{n\to\infty} \frac{e^{n+1}}{1+e^{2(n+1)}} \cdot \frac{1+e^{2n}}{e^n}$

$= \lim_{n\to\infty} e\left(\frac{1+e^{2n}}{1+e^{2n+2}}\right)$

$= e \cdot \lim_{n\to\infty} \frac{\cancel{2}e^{2n}}{\cancel{2} \cdot e^{2n+2}}$ (L'Hopital)

$= e \cdot \lim_{n\to\infty} \frac{e^{2n}}{e^2 \cdot e^{2n}}$

$= e \cdot \frac{1}{e^2}$

$= \frac{1}{e} < 1$

converges by Ratio Test

Compare to $\sum \frac{1}{e^n}$

Since $\frac{e^n}{1+e^{2n}} < \frac{1}{e^n}$

and $\sum \frac{1}{e^n}$ converges

so does $\sum \frac{e^n}{1+e^{2n}}$

by DCT

(16) $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n+2)(n^2+5)}$ Compare to $\sum \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{5n^3-3n}{n^5+\cdots} \cdot \frac{n^2}{1}$$

$$= \lim_{n\to\infty} \frac{5n^5-3n^3}{n^5+\cdots} = 5$$

$\therefore$ both converge or diverge and since $\sum \frac{1}{n^2}$ converges so does $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n+2)(n^2+5)}$ by LTC

(17) $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$

$$\lim_{n\to\infty} \frac{3^{n-1}+1}{3^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3}\cdot 3^n+1}{3^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3}+\frac{1}{3^n}}{1}$$

$$= \frac{1}{3} \neq 0$$

$\therefore$ diverges by n^{th} term test